Econ 4130

2005 H

Exercises for seminar week 46 (group I) and week 47 (group II)

Supplementary Exercise 7

a. Suppose that $X_1, X_2, \ldots, X_m$ are independent and poisson distributed, where $X_i \sim \text{pois}(\lambda_i)$ for $i = 1, 2, \ldots, m$. Show that $N = X_1 + X_2 + \cdots + X_m \sim \text{pois}(\lambda)$, where $\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_m$. [**Hint:** Show that the mgf of X is a poisson mgf.]

b. Let $X_1, X_2, \ldots, X_m$ and $N = X_1 + X_2 + \cdots + X_m$ be as in **a.** Show that the joint conditional distribution of $X_1, X_2, \ldots, X_m$, given $N = n$, is multinomial $(n, p_1, \ldots p_m)$ where the cell probabilities are given by $p_j = \dfrac{\lambda_j}{\lambda_1 + \lambda_2 + \cdots + \lambda_m}$. [**Hint:** Note that

$$f(x_1, x_2, \ldots, x_m \mid n) = P(X_1 = x_1 \cap \cdots \cap X_m = x_m \mid N = n) = \frac{P(X_1 = x_1 \cap \cdots \cap X_m = x_m)}{P(N = n)}$$

where $x_1, x_2, \ldots, x_m$ satisfy $x_1 + x_2 + \cdots + x_m = n$. Explain the last equality.]

Rice Chapter 8

No. 2

Rice Chapter 9

No. 18
No. 28 (Hint: Read section 9.7 in Rice).
No. 31 [Don't pay too much attention to the answer at the end of the book.]